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MEMORANDUM REPORT NO. 2655

QUASI-STATIC COMPRESSION STRESS-STRAIN  
CURVES--IV, 2024-T3510 AND 6061-T6  
ALUMINUM ALLOYS

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August 1976

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## I. INTRODUCTION

The quasi-static compression tests reported herein were conducted in connection with the Core Materials Program of the Solid Mechanics Branch of the Terminal Ballistics Laboratory.

The purpose of the Core Materials Program is to characterize the mechanical behavior of armor and armor penetrators. This characterization should prove useful to designers of armored vehicles and projectiles, and will provide valuable input data for computer codes modeling penetration processes.

This report presents the results of quasi-static compression tests on two aluminum alloys; 2024-T3510, and 6061-T6. These results include the yield strength, average stress-strain curve, Poisson's ratio, and Young's moduli for each aluminum alloy.

These two materials are the fourth and fifth in a series<sup>1,2,3</sup> which includes seven steel and seven aluminum alloys\*. The results of other tests will follow when completed.

## II. TEST PROCEDURES

The testing apparatus, procedures and data reduction regimen have been reported previously<sup>1</sup>. The test specimens of each material were machined from one-inch diameter rods of commercial purity. Six test specimens of each material were prepared as right circular cylinders, 9.5mm in diameter and 28.6mm long. Samples of both materials were chemically analyzed at the Frankford Arsenal. The temperature for the tests was 22°C.

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<sup>1</sup>E. A. Murray, Jr. and J. H. Suckling, BRL MR 2399, "Quasi-Static Compression Stress-Strain Curves--I, 1066 Steel", Ballistics Research Laboratories, APG, MD., January 1974. AD 922 704 L.

<sup>2</sup>E. A. Murray, Jr., BRL MR 2589, "Quasi-static Compression Stress-Strain Curves--II, 7039 Aluminum," Ballistics Research Laboratories, APG, MD. February 1976. AD #B009646L.

<sup>3</sup>Ralph F. Benck and E. A. Murray, Jr., BRL MR 2480, "Quasi-Static Compression Stress-Strain Curves--III, 5083-H131 Aluminum", Ballistics Research Laboratories, APG, MD. May 1975. AD B00 4159 L.

\*Steel Alloys: 1020, 1066, 4145, 4160, 4340, Bearcat, and RHA. Aluminum Alloys: 1100F, 2024-T3510, 5083-H131, 6061-T6, 7039, 7075, and 7475.

### III. RESULTS

The average engineering stress-strain curves for six specimens each of 2024-T3510 and 6061-T6 aluminum alloy are shown in Figures 1 and 2, respectively. The vertical error bands in the figures are the variations of stress of plus and minus one standard deviation. The tests were terminated upon failure of one of the strain gages. Table I shows the maximum strain attained prior to gage failure. The curves presented in Figures 1 and 2 are the averages from at least two tests.

Figures 3 and 4 present longitudinal and circumferential stress-strain relationships for a representative specimen each of the 2024-T3510 and 6061-T6 aluminum alloy material.

The curves labeled "longitudinal" in Figures 3 and 4 indicate both the individual response of each of two diametrically opposing gages as well as their average. The average value is the longitudinal strain in the specimen at any load (~stress), and divergence from this average is indicative of the amount of bending present. The near coalescence of the curves for individual gages with their average in each test demonstrates the high degree of axiality maintained throughout these compression tests.

Poisson's ratios as a function of strain for these two alloys are similar; examples of Poisson's ratio up to one and up to five percent strain are shown in Figures 5 and 6, respectively. Poisson's ratio for both alloys is 0.32 in the elastic region and approaches 0.5 as the material becomes more and more plastic.

The average yield strength, Young's modulus and Poisson's ratio for the 2024-T3510 and 6061-T6 alloys are presented in Table II. The number within the parentheses is the standard deviation based on six tests of each alloy. The yield strength is defined as that stress at which the specimens deviated 0.2 percent from proportionality of stress to strain<sup>4</sup>.

The results of chemical analyses of samples of both alloys are shown in Table III.

---

<sup>4</sup>Taylor Lyman, Ed., Metals Handbook, 1948 Edition, The American Society for Metals, Cleveland, Ohio, p. 16.

TABLE I

## MAXIMUM STRAIN PRIOR TO GAGE FAILURE

SPECIMEN	2024-T3510 %	6061-T6 %
1	7.26	8.54
2	7.37	4.68
3	9.37	6.04
4	9.76	4.75
5	8.31	4.87
6	3.39	8.12

TABLE II

MEASURED MATERIAL PROPERTIES  
OF 2024-T3510 AND 6061-T6 ALUMINUM ALLOYS AT 22°C

PROPERTY	2024-T3510	6061-T6
Average Yield Strength, MPa <S.D.> *	444 <5.1>	267 <2.3>
Young's modulus, GPa <S.D.> *	76.1 <0.9>	72.2 <0.7>
Poisson's ratio	0.321	0.320
Hardness, BHN	148	95

\*S.D. = Standard Deviation

TABLE III

CHEMICAL ANALYSIS OF 2024-T3510 AND  
6061-T6 ALUMINUM ALLOYS\*

ELEMENT	WEIGHT PERCENT	
	2024-T3510	6061-T6
Copper	4.20	0.2/0.4
Silicon	0.1/0.2	0.4/0.8
Iron	0.2/0.4	0.15/0.35
Manganese	0.4/0.8	0.06
Zinc	0.05/0.15	<0.1
Magnesium	1.50	1.11
Titanium	<0.05	<0.05
Chromium	<0.03	0.10/0.25
Nickel	<0.02	<0.01
Tin	None detected	None detected
Lead	<0.05	0.05, <0.05
Aluminum	Remainder	Remainder

\*Analysis by Frankford Arsenal, Materials Laboratory, Technical Support Directorate. Spectroscopic Analysis.

An analytical relationship between Poisson's ratio and strain over the range depicted in Figure 6 has not been documented, or at least no references to such a relationship have been found. While the initial (elastic) and final (0.5) values have been amply discussed in texts and the literature, the form for Poisson's ratio in the region between onset of yielding and final values has never been formalized. Yet in principle, if the claims of classical plasticity theory concerning incompressible plastic deformations are correct, the transition in Poisson's ratio from elastic value to asymptotic large plastic value should be orderly and predictable from the longitudinal stress-strain curve. In an effort to add to the understanding of information such as that presented in Figure 6, an analytical relationship between Poisson's ratio and longitudinal strain, based on incompressible plasticity, is developed in the Appendix. It is expressed as:

$$\nu_{\text{apparent}} = \frac{\hat{\sigma}}{\epsilon_{xx}(\hat{\sigma})} \left( \frac{\nu_e - 1/2}{E} \right) + \frac{1}{2} \quad (1)$$

where

$\nu_{\text{apparent}}$  = Poisson's ratio in plastic region

$\hat{\sigma}$  = longitudinal stress

$\epsilon_{xx}(\hat{\sigma})$  = longitudinal strain at  $\hat{\sigma}$  stress

$\nu_e$  = Poisson's ratio determined from static tests via extrapolation to zero strain.

E = Young's modulus determined from static test via "best fit" in linear elastic region.

In developing (1) it has been assumed that the "plastic" component of strain is incompressible, viz.

$$\frac{\epsilon_{\theta\theta}(\hat{\sigma}) - \frac{\hat{\sigma}\nu_e}{E}}{\epsilon_{xx}(\hat{\sigma}) - \frac{\hat{\sigma}}{E}} = 1/2 \quad (2)$$

All terms in Equation 2 are as defined for Equation 1, and  $\epsilon_{\theta\theta}(\hat{\sigma})$  is the circumferential strain at longitudinal stress  $\hat{\sigma}$ .

$\nu_{\text{apparent}}$  and  $\nu_{\text{predicted}}$  are plotted in Figure 7 versus  $\epsilon_{xx}$  for one sample of 2024-T3510 alloy. For these calculations the measured values of E and  $\nu$  (76.588 GPa and 0.323 respectively) for this particular test were used and not the average values shown in Table II.  $\nu_{\text{apparent}}$  is Poisson's ratio calculated from the ratio of the experimentally measured longitudinal and circumferential strains.

Figure 7 shows that expression (1) is an accurate predictor of Poisson's ratio for the rapidly ascending portion of the curve. For the remainder of Figure 7 the curves separate with the maximum separation being about five percent. The five other samples of 2024-T3510 and those of 6061-T6 that are reported herein yielded curves similar to those shown in Figure 7 with the difference between  $\nu_{\text{apparent}}$  and  $\nu_{\text{predicted}}$  at strains greater than two percent being in the order of plus or minus six percent.

In lieu of assuming an incompressible plastic component of the deformation and linear work hardening, one may in fact, take the other point of view and use the test data of this report to compute the material compressibility as a function of stress or strain. In this way, a separate check is made on the final assumptions of the analytical development in the Appendix.

Continue to assume homogeneous deformation and stress states as put forth in the Appendix. Consistent with notation introduced there, the initial and final volumes of a cylinder are  $V_0 = \pi r_0^2 l_0$  and  $V_1 = \pi(r')^2 (1+\epsilon_{xx}) l_0 = \pi r_0^2 (1+\epsilon_{\theta\theta})^2 (1+\epsilon_{xx}) l_0$ . Hence one easily has for the compressibility

$$\frac{\Delta V}{V_0} = \frac{V_1 - V_0}{V_0} = \left[ (1 + \epsilon_{\theta\theta})^2 (1 + \epsilon_{xx}) - 1 \right] \quad (3)$$

The Poisson's ratio vs. strain data of Figures 5, 6, and 7 coupled with Appendix Equation A-4 may be used to recover the observed variable  $\epsilon_{\theta\theta}$  as a function of  $\epsilon_{xx}$ . Equation (3) expresses the compressibility  $\frac{\Delta V}{V_0}$  implicitly as a function of stress; the functional form may be made an explicit function of  $\sigma$  by means of the experimentally determined stress-strain relation  $\epsilon = \epsilon(\hat{\sigma})$ . Thus one may plot  $\frac{\Delta V}{V_0}$  as a function of  $\sigma$ , and this should be the most instructive manner in which to view this interdependence. For, a consequence of the development in the Appendix is that only the current value of the elastic component of strain (in the elastic-plastic decomposition) contributes to volume change. Since this is taken to be linearly related to longitudinal stress, equal stress increments should cause equal volume change increments.

Figure 8 shows a plot of the compressibility  $\frac{\Delta V}{V_0}$  as a function of stress, calculated by means of Equation 3 and the stress-strain curve for the same test that was used to derive Figure 7. The compressibility is nearly linear from the origin to a point whose stress value is 465 MPa. Note the radical departure in the compressibility curve from the linear one at 465 MPa, as well as the first detectable departure at 425 MPa. Interestingly enough, the behavior beyond 465 MPa is nearly linear also, but at a much reduced modulus of compressibility. It is also noteworthy that the strain corresponding to 465 MPa is well beyond the "knee" of the stress-strain curve of Figure 1, and corresponds to a strain of 1.10 percent. The proportional limit is closer to 425 MPa, the first detectable departure point in Figure 8, with corresponding strain 0.59 percent. These two observations are consistent with the extremely close agreement between apparent (measured) and predicted values of Poisson's ratio up to 1.10 percent strain in Figure 7. Although the differences beyond this strain were within  $\pm 6$  percent, they are now known to be related to the radical change in compressibility at 1.10 percent longitudinal strain.

At this point, it is conjecture as to the causes of (1) the radical change in material compressibility noted here and (2) its delayed occurrence well beyond the proportional limit. The results, however, are not inconsistent with empirical multi-stress component plasticity theories of Bell<sup>5</sup> as to regions of onset of total plasticity and the transition strains. Further investigations will be reported at a later date.

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<sup>5</sup>James F. Bell, BRL CR 250, "A New, General Theory of Plasticity for Structural Metal Alloys", Ballistics Research Laboratories, APG, MD, July 1975. AD #A014192L.

#### IV. CONCLUSIONS

Quasi-static compression tests were made on 2024-T3510 and 6061-T6 aluminum alloys. The data acquired from these tests have been reduced and are in a form readily applicable for users.

It is evident from the reproducibility of the data, that the results presented are an accurate, partial description of the elastic and plastic properties of 2024-T3510 and 6061-T6 aluminum alloys.

An analytical expression has been developed that accurately predicts the behavior of Poisson's ratio as a function of strain in the strain region beyond the proportional limit.

Well beyond the proportional elastic limit, a radical change occurs in the compressibility of each of these two alloys, and is the point of demarcation where the above analytical prescription begins to develop errors of  $\pm 6$  percent. The compressibility may relate to newer empirical constitutive equations developed by Bell (see Reference 5), but such a connection is speculative at this point.

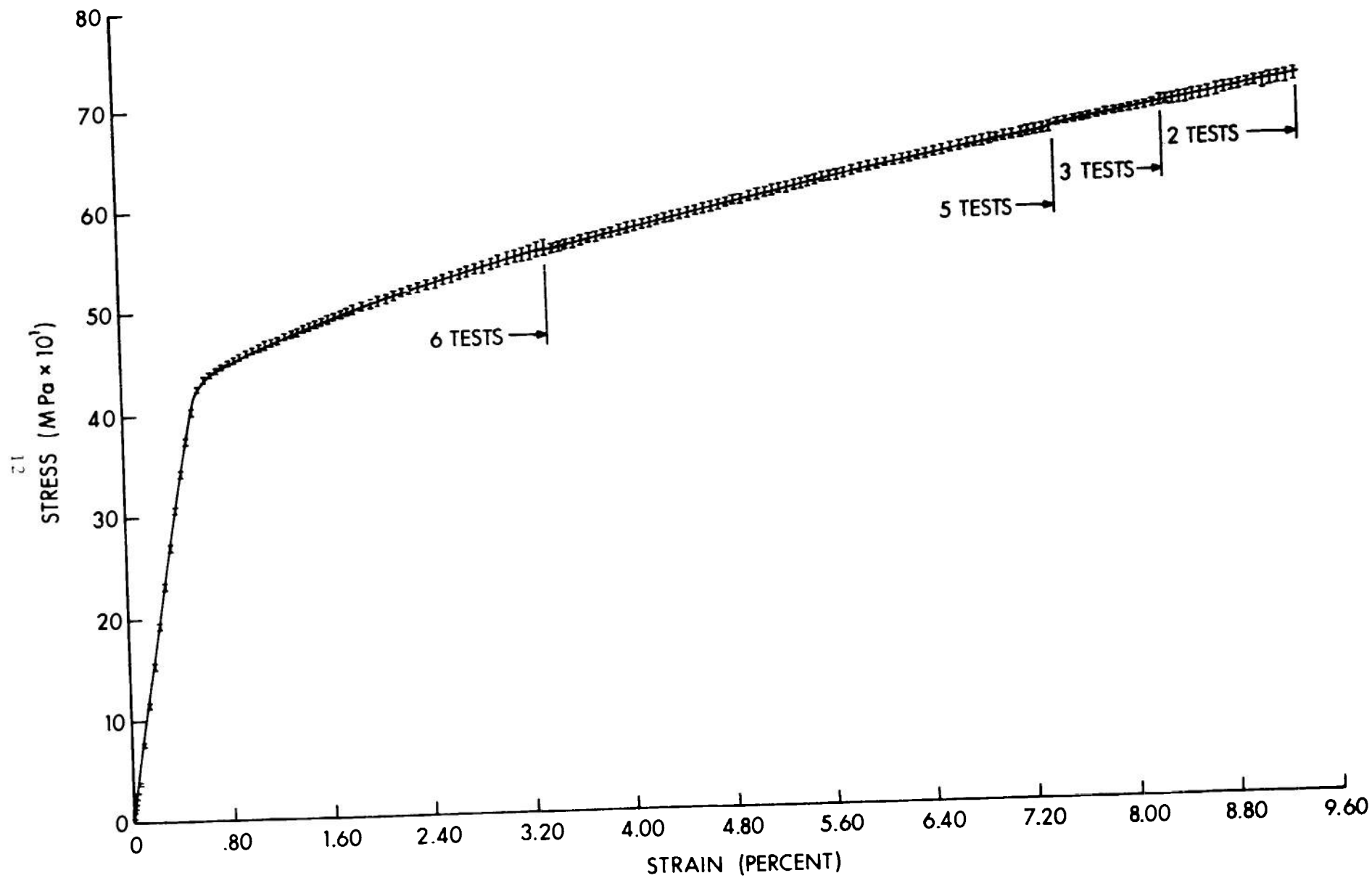


Figure 1. Average Stress - Strain Curve for Compression Test of 2024 T 3510 Aluminum.

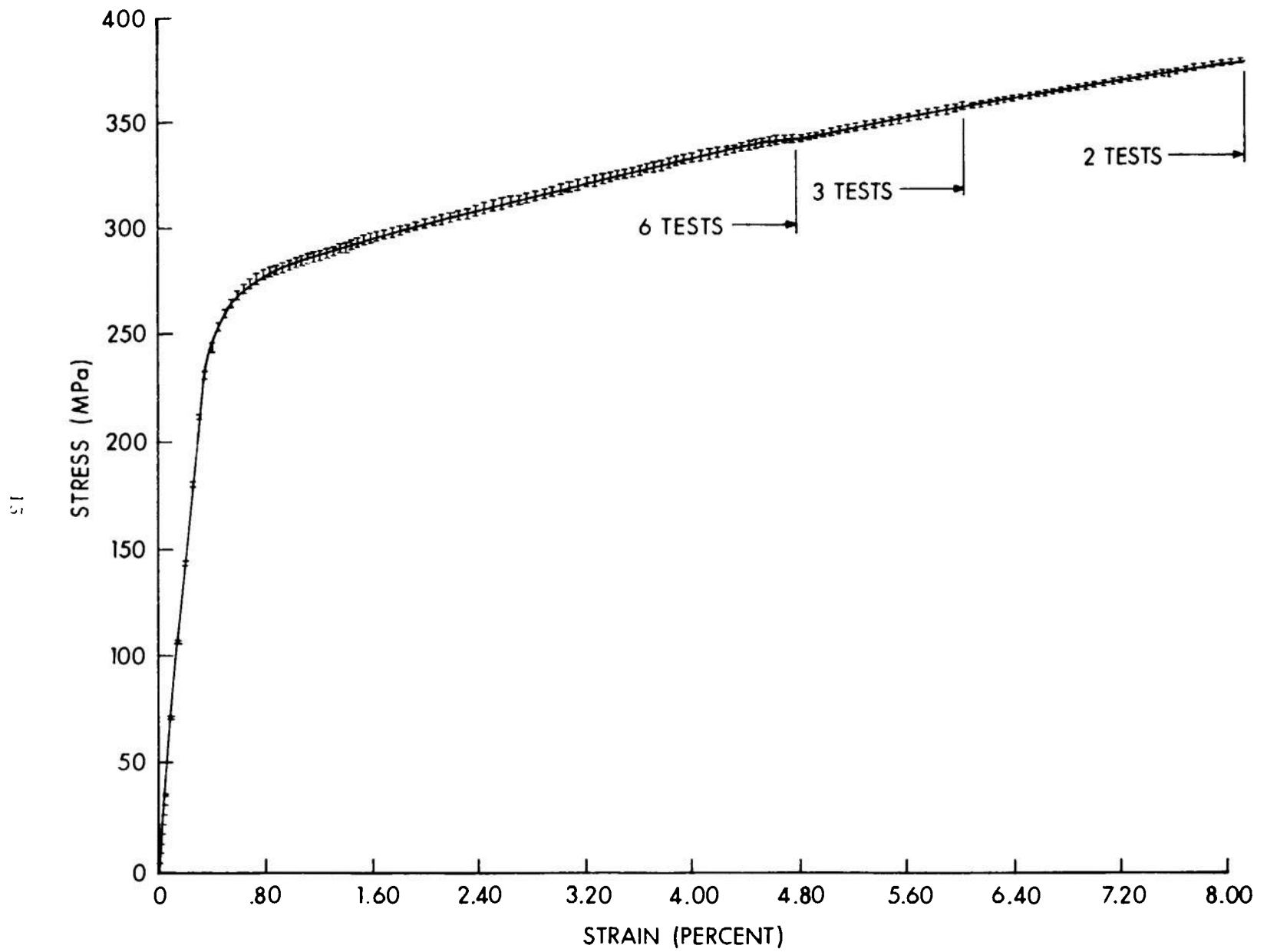


Figure 2. Average Stress-Strain Curves for Compression Test of 6061 T6 Aluminum.

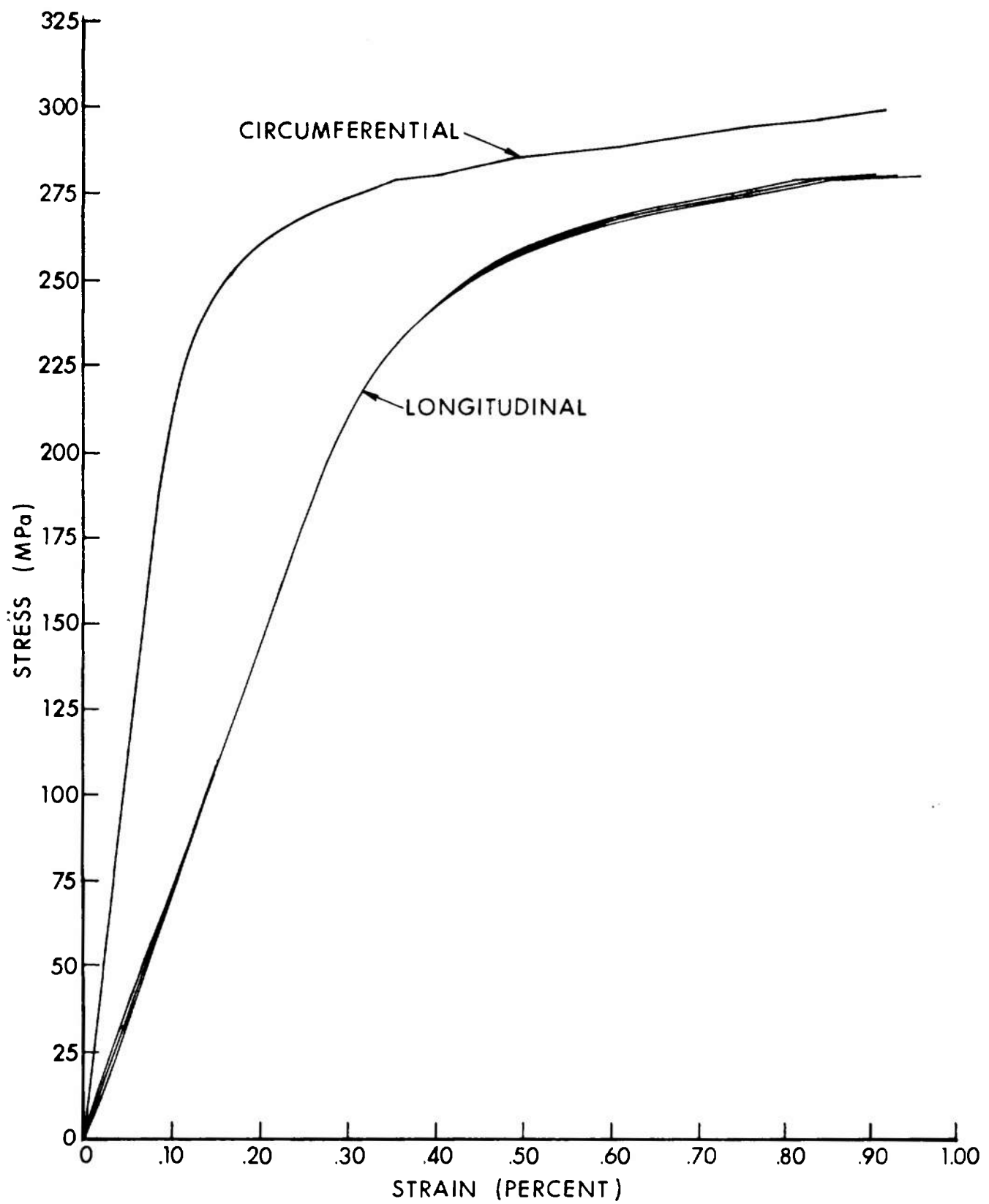


Figure 4. Stress-Strain Curves for One Specimen of 6061 T6 Aluminum.

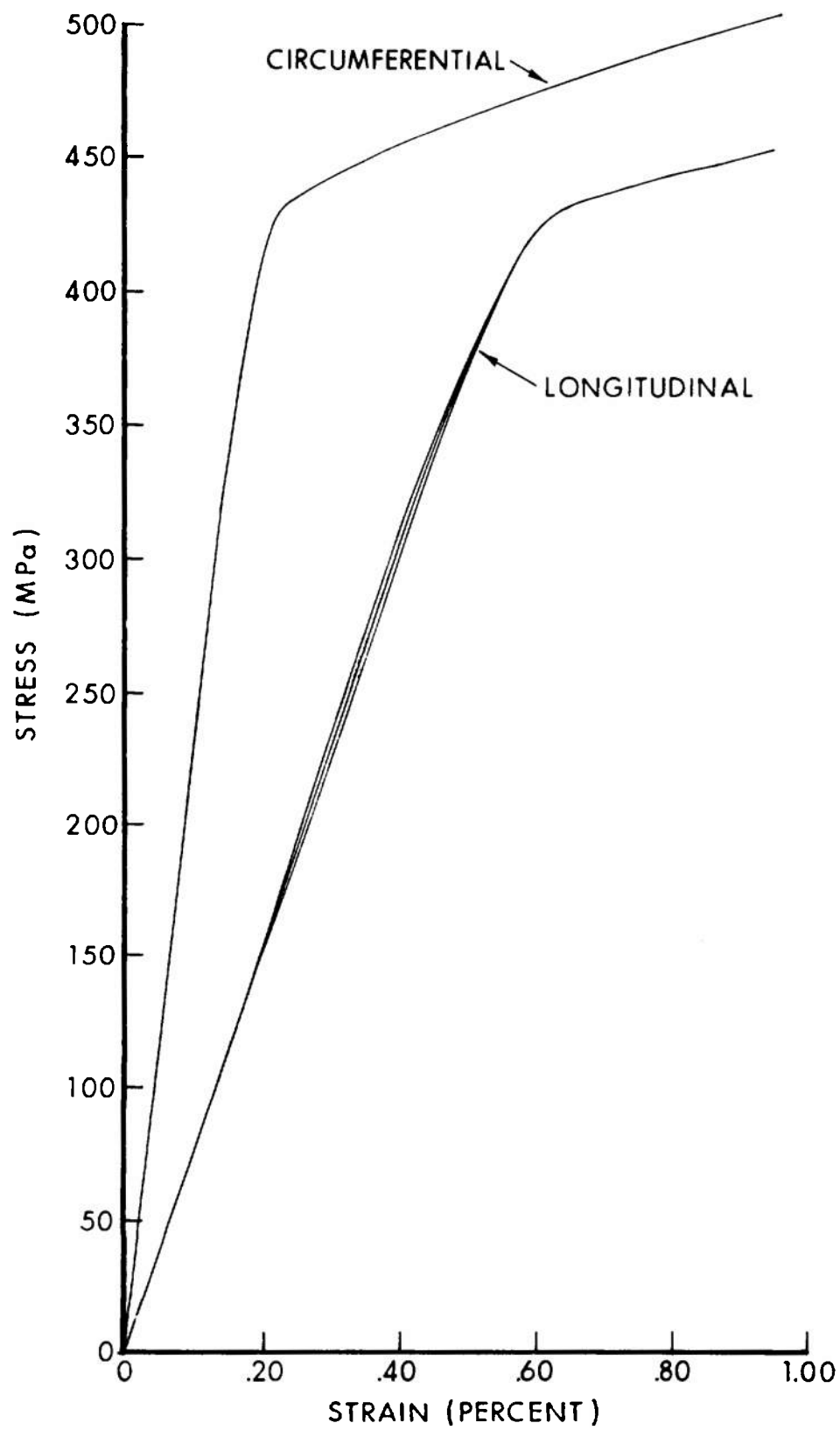


Figure 3. Stress -Strain Curves for One Specimen of 2024 T3510 Aluminum

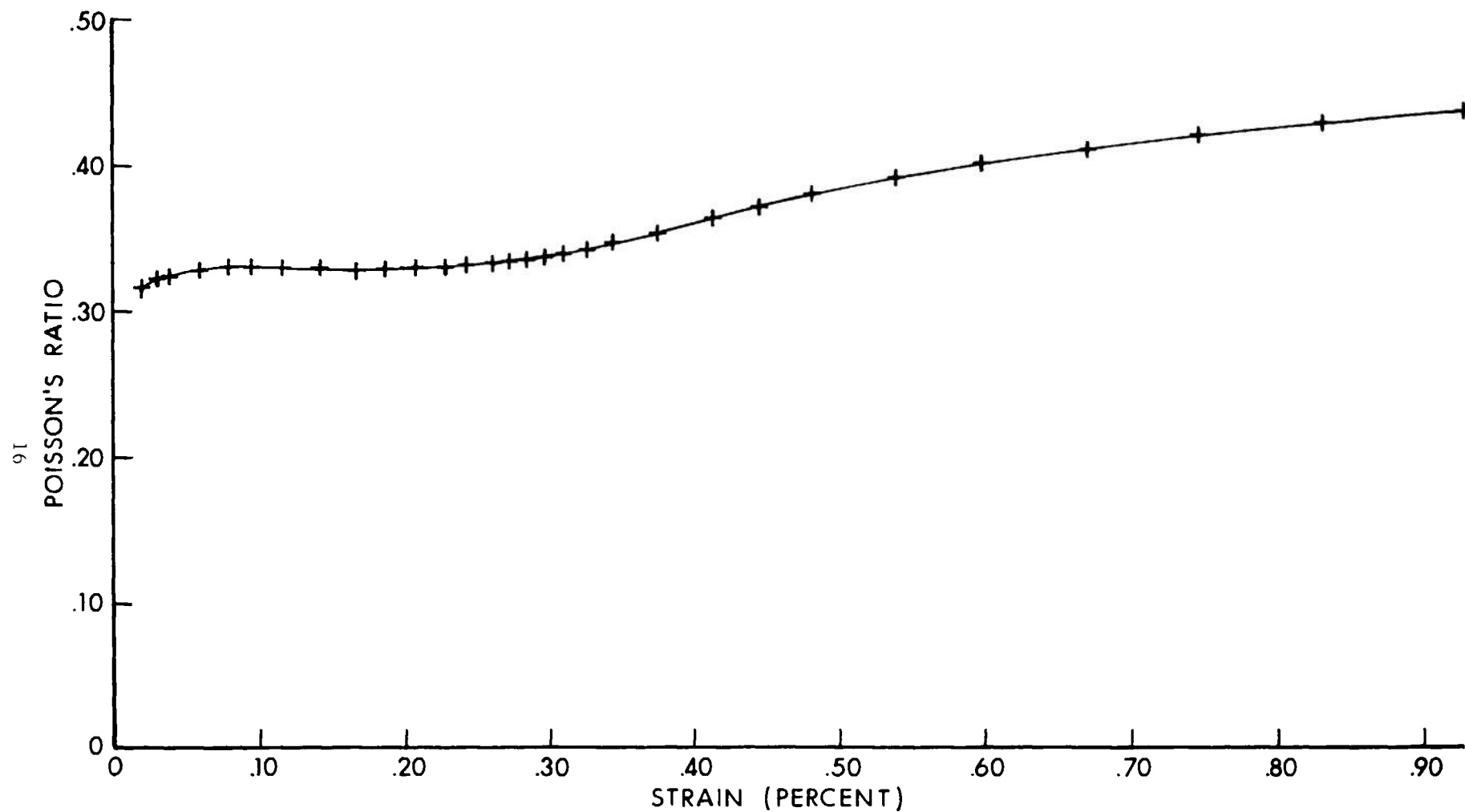


Figure 5. Poisson's Ratio as a Function of Strain up to One Percent Maximum Strain for 6061 T6 Aluminum.

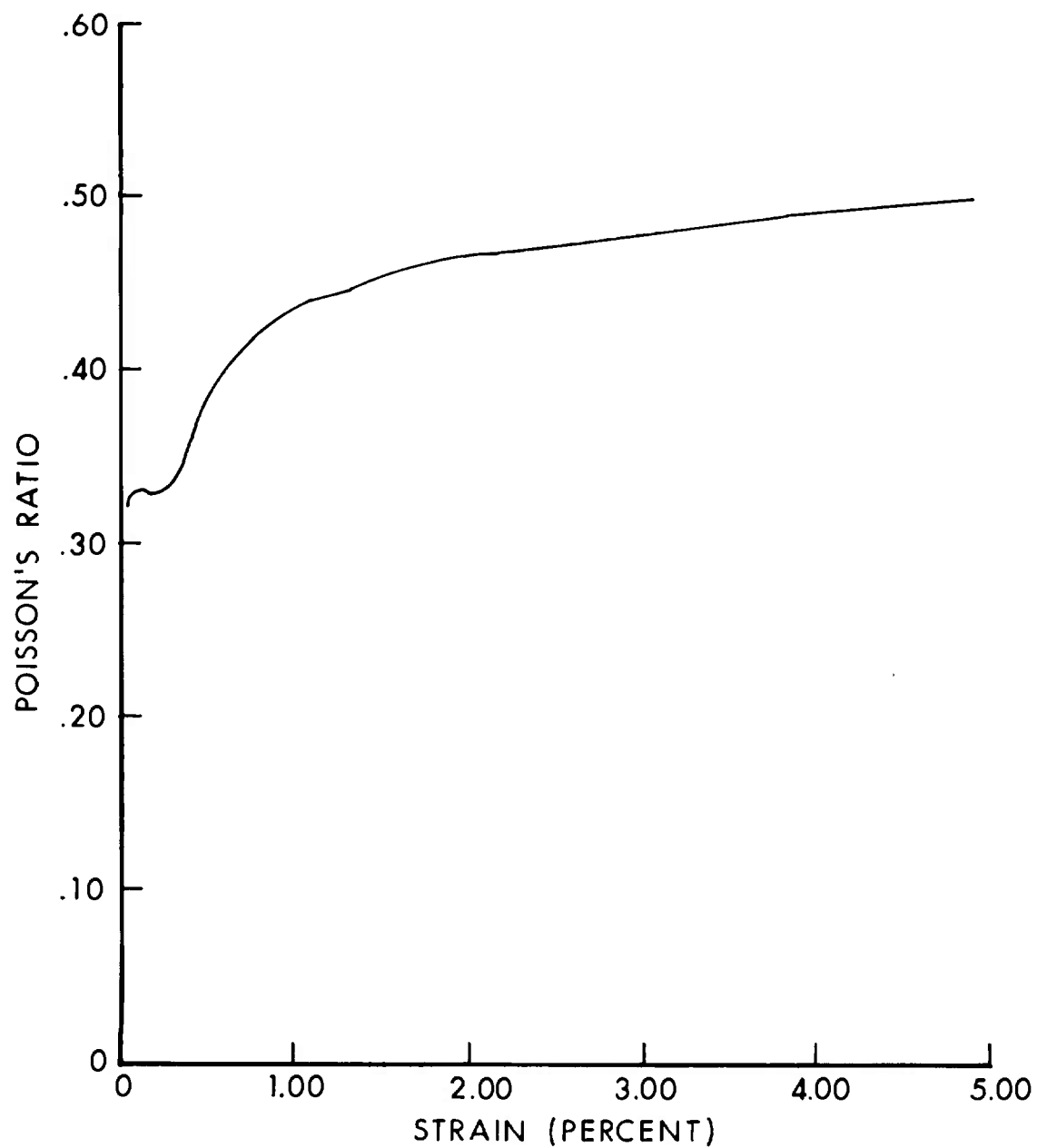


Figure 6. Poisson's Ratio as a Function of Strain up to Five Percent Maximum Strain for 6061 T6 Specimen.

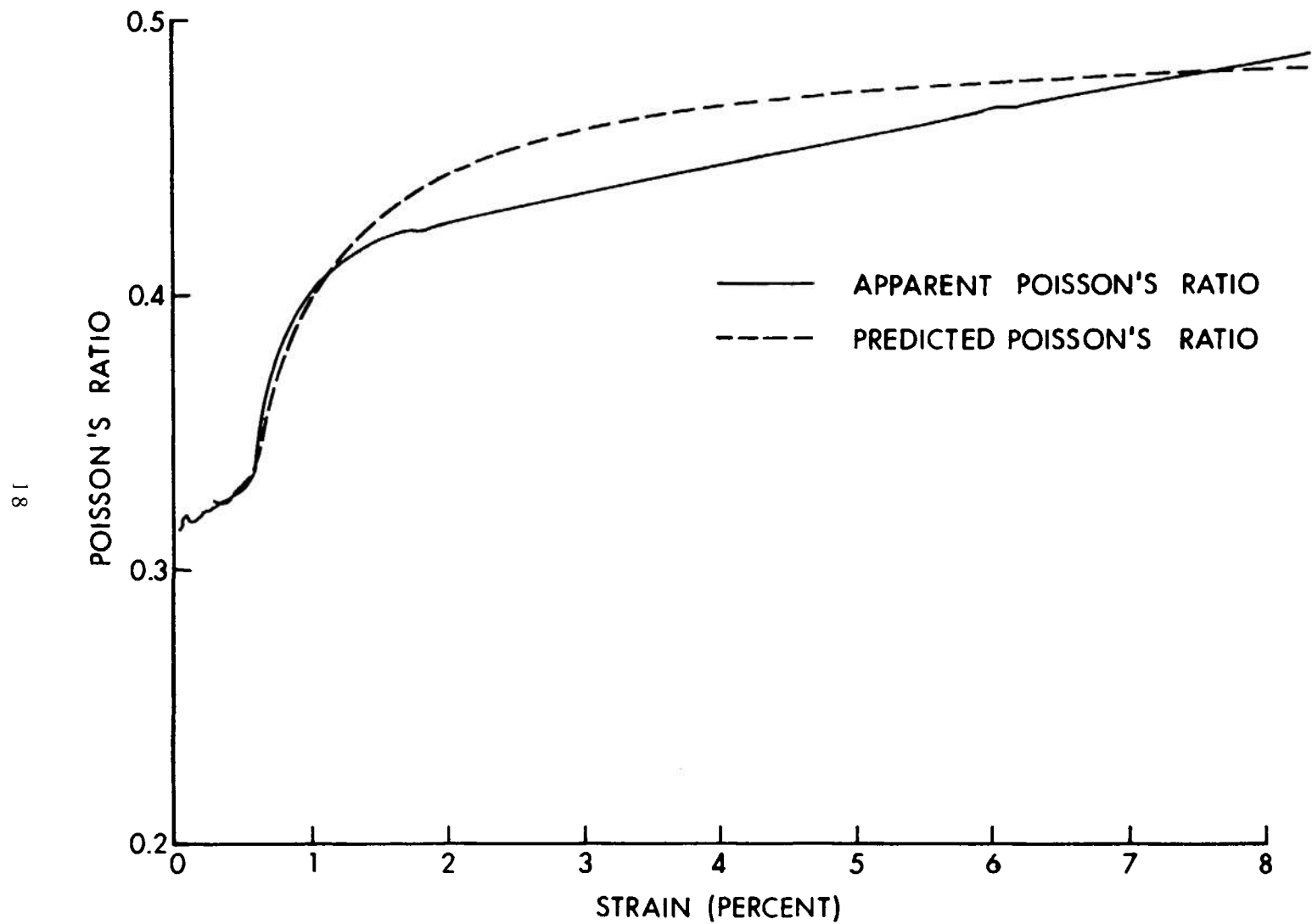


Figure 7. Predicted and Apparent Poisson's Ratio as a Function of Strain for 2024 T3510 Specimen.

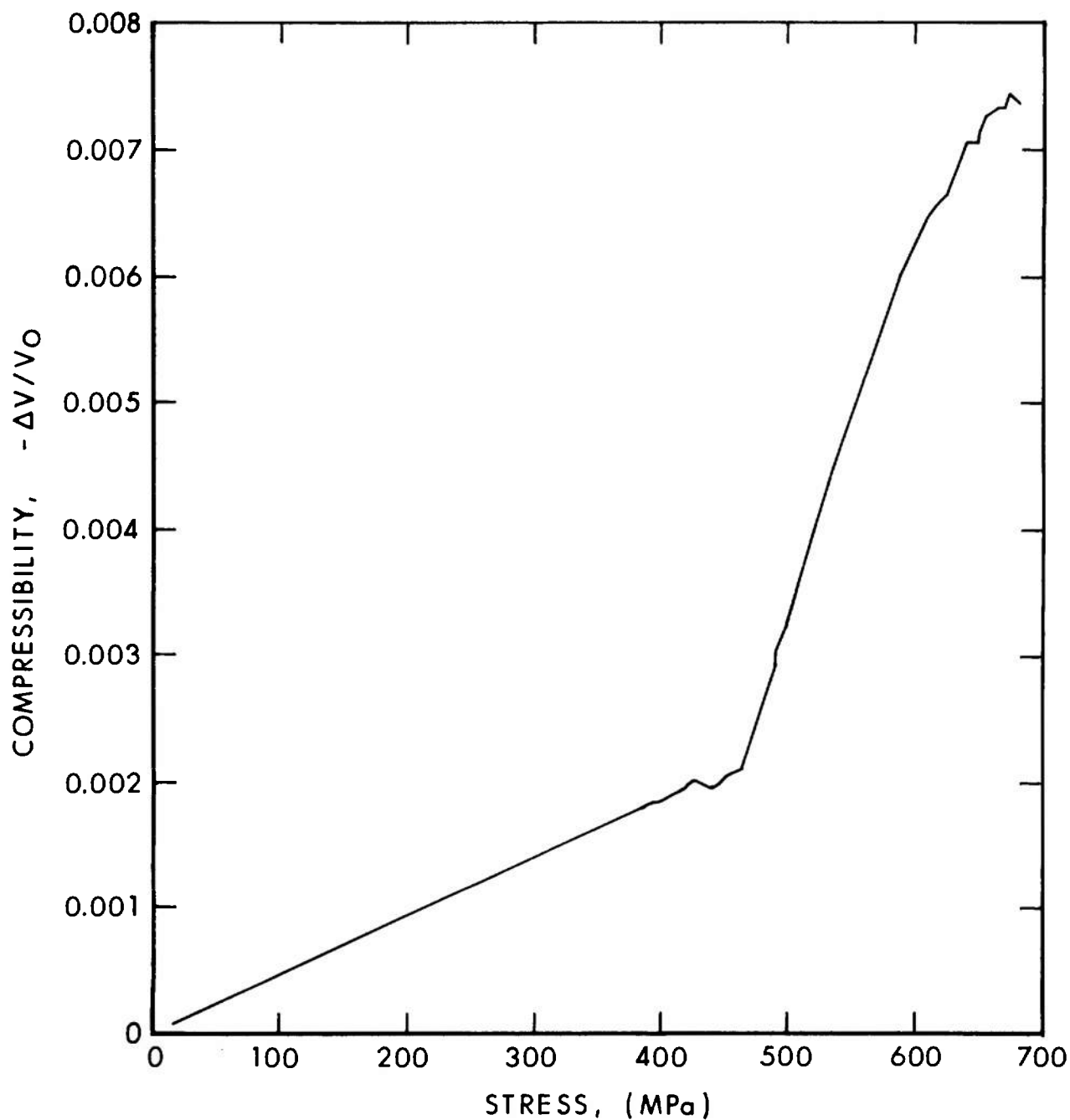


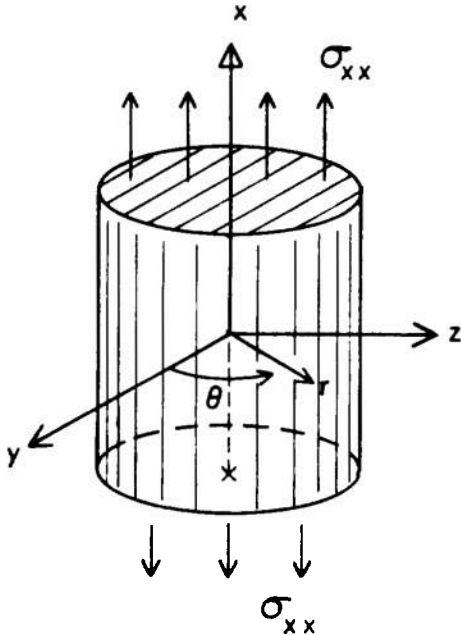
Figure 8. Compressibility as a Function of Stress for 2024 - T3510 Specimen.

## APPENDIX

### POISSON'S RATIO AS A FUNCTION OF STRAIN

## APPENDIX

An expression for the apparent Poisson's ratio in the strain region beyond the linear elastic limit and preceding large plastic deformations ( $> 10\%$ ) will be developed. Consider the homogeneous deformation of a right circular cylinder of material as should be experienced in laboratory uni-axial tension or compression testing. Assume that plane sections normal



to the  $x$  axis in the accompanying sketch remain plane, and that the strain fields are homogeneous in  $x$  and  $r$ , and independent of  $\theta$ . The apparent Poisson's ratio,  $\nu_{app}$ , in such an experiment is defined as the ratio of radial contraction to longitudinal extension, normalized to strain measures. That is, if the cylinder sketched is of initial length  $l_0$  and radius  $r_0$ , and the longitudinal extension is  $u_x$  and the accompanying radial motion is  $u_r$  (positive if outward), then

$$\nu_{app} = - \frac{u_r/r_0}{u_x/l_0} \quad (A-1)$$

The longitudinal "engineering" strain  $\epsilon_{xx}$  is recognized as the denominator  $u_x/l_0$ , and would be the quantity measured by a longitudinal strain gage. The quantity in the numerator,  $u_r/r_0$ , is the "hoop" engineering strain  $\epsilon_{\theta\theta}$ . Let  $r_0$  be the initial radius of the cylindrical surface, and  $r' = r_0 + u_r$  the radius after some homogeneous deformation. Then the hoop engineering strain would be circumferential strain

$$\epsilon_{\theta\theta} = \frac{2\pi r' - 2\pi r_0}{2\pi r_0} = \frac{u_r}{r_0} \quad (A-2)$$

This is the quantity which would be measured by a strain gage oriented in the "hoop" direction. Thus, the apparent Poisson's ratio is determined by:

$$\nu_{app} = - \frac{\epsilon_{\theta\theta}}{\epsilon_{xx}} \quad (A-3)$$

A typical stress-strain curve for materials of interest would have the form represented in the second sketch. Plotted also on this sketch is the negative value of the hoop strain  $\epsilon_{\theta\theta}$ . Since



where  $\epsilon_{xx}^{(e)}$  and  $\epsilon_{\theta\theta}^{(e)}$  are the strains respectively at the points M and N, the proportional limit. This is, of course, the elastic value of Poisson's ratio. In the linear elastic region the connection between stress and longitudinal strain is

$$\sigma_{xx} = E \epsilon_{xx} \quad (A-6)$$

where E is the usual Young's modulus. From the sketch and Equation A-5, a connection can also be written between stress and hoop strain in the linear elastic region as

$$\sigma_{xx} = E^* \epsilon_{\theta\theta} = E^* \nu_e \epsilon_{xx}, \quad (A-7)$$

where  $E^*$  is an apparent modulus. Comparing Equations A-6 and A-7 yields

$$E^* = \frac{E}{\nu_e} \quad (A-8)$$

Make strain decompositions into components, so that the total strain is the sum of an elastic and plastic component, expressed as

$$\epsilon_{xx} = \epsilon_{xx}^{(e)} + \left( \epsilon_{xx} - \epsilon_{xx}^{(e)} \right). \quad \text{Equation A-4 may be written}$$

$$\nu_{app} = \frac{\epsilon_{\theta\theta}^{(e)} + \left( \epsilon_{\theta\theta} - \epsilon_{\theta\theta}^{(e)} \right)}{\epsilon_{xx}^{(e)} + \left( \epsilon_{xx} - \epsilon_{xx}^{(e)} \right)} \quad (A-9)$$

But now assume plastic work hardening. This means that the elastic component is the elastic strain which would be recovered if unloading should occur from a stress  $\hat{\sigma}$ , even though it is not actually done. Hence, with Equation A-6 and A-7, Equation A-9 becomes

$$\nu_{app} = \frac{\frac{\hat{\sigma}}{E^*} + \left( \epsilon_{\theta\theta}(\hat{\sigma}) - \frac{\hat{\sigma}}{E^*} \right)}{\frac{\hat{\sigma}}{E} + \left( \epsilon_{xx}(\hat{\sigma}) - \frac{\hat{\sigma}}{E} \right)} \quad (A-10)$$

Equations (A-8) and (A-10) may be combined to yield

$$\nu_{app} = \frac{\frac{\hat{\sigma} \nu_e}{E} + \left( \epsilon_{\theta\theta}(\hat{\sigma}) - \frac{\hat{\sigma} \nu_e}{E} \right)}{\frac{\hat{\sigma}}{E} + \left( \epsilon_{xx}(\hat{\sigma}) - \frac{\hat{\sigma}}{E} \right)} \quad (A-11)$$

The expression is often heard, "Poisson's ratio for plastic deformation is equal to one-half". Assuming an incompressible homogeneous deformation of the cylinder sketched previously, and if the volume before deformation is  $V_0 = \pi r_0^2 l_0$  and following deformation is  $V_1 = \pi (r')^2 (1 + \epsilon_{xx}) l_0$ , one finds that setting  $V_0 = V_1$  yields

$$r_0^2 = (r')^2 (1 + \epsilon_{xx})$$

or that

$$\begin{aligned}\frac{r'}{r_0} &= 1 + \frac{u_r}{r_0} = (1 + \epsilon_{xx})^{-1/2} \\ &= 1 - \frac{1}{2} \epsilon_{xx} + \frac{3}{8} \epsilon_{xx}^2 - \dots\end{aligned}\quad (A-12)$$

Thus, with Equation A-2, Equation A-12 becomes, for an incompressible deformation

$$\epsilon_{\theta\theta} = -\frac{1}{2} \epsilon_{xx} (1 - \frac{3}{4} \epsilon_{xx} + \dots) \quad (A-13)$$

Comparing Equations A-13 and A-3, one sees that the apparent Poisson's ratio for a totally incompressible deformation is approximately 1/2, to a first order error correction of  $O(3/4 \epsilon_{xx})$ . This means that for a 4 percent longitudinal strain, Poisson's ratio differs from 1/2 by 3 percent. Since longitudinal strains of this order are discussed in this report, this level of error shall be accepted. However, Poisson's ratio is not 1/2 in the elastic region\*, so that the assumption of total incompressibility from the initial state is inconsistent with reality.

Assume now, in the expression developed into Equation A-11, that the plastic component of the deformation is incompressible. Thus, within an error band of approximately 3 percent, by Equation A-13 one has

$$\left( \epsilon_{\theta\theta}(\hat{\sigma}) - \frac{\hat{\sigma} \nu_e}{E} \right) = \frac{1}{2} \left( \epsilon_{xx}(\hat{\sigma}) - \frac{\hat{\sigma}}{E} \right) \quad (A-14)$$

With Equation A-14, Equation A-11 becomes

$$\nu_{\text{apparent}} = \frac{\frac{\hat{\sigma} \nu_e}{E} + \frac{1}{2} \left( \epsilon_{xx}(\hat{\sigma}) - \frac{\hat{\sigma}}{E} \right)}{\epsilon_{xx}(\hat{\sigma})} \quad (A-15)$$

which may be rearranged to

$$\nu_{\text{apparent}} = \frac{\hat{\sigma}}{\epsilon_{xx}(\hat{\sigma})} \left( \frac{\nu_e - \frac{1}{2}}{E} \right) + \frac{1}{2} \quad (A-16)$$

In Equation A-16,  $\nu_e$  and  $E$  are experimentally determined from linear elastic region data for the material under test,  $\hat{\sigma}$  and  $\epsilon_{xx}(\hat{\sigma})$  are the longitudinal stress and longitudinal strain at stress  $\hat{\sigma}$ , and  $\nu_{\text{apparent}}$  is the predicted Poisson's ratio at these values. Equation A-16 is equivalent to Equation 1 of the text.

\*See, e.g., Table II of this report.

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